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Superfield Realizations of Lorentz Violation[*]

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Abstract

Lorentz-violating extensions of the Wess-Zumino model have been formulated in superspace. The models respect a supersymmetry algebra and can be understood as arising from suitably modified superspace transformations.

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I. INTRODUCTION

Spacetime symmetries have always played a central role in particle physics. Some of these symmetries like the Poincaré symmetry are taken to be exact, while other symmetries like supersymmetry are assumed to be broken. From the perspective of experimental physics it appears that supersymmetry is a badly broken symmetry, so much so that there has to date been no direct evidence for it. However viewed from the Planck scale, electroweak-scale supersymmetry is almost exact. Indeed much theoretical research has been devoted to accounting for the ratio of the supersymmetry breaking scale to the Planck scale. From this point of view, it seems that if one accepts the possibility that the a spacetime symmetry such as supersymmetry is broken in a very small manner, that one should entertain the possibility that the remaining spacetime symmetries are also broken to a very small extent even though there is no experimental evidence for it.

As the possibility that Lorentz and CPT violation might occur in fundamental theories has become more apparent, there has been interest in finding supersymmetric theories with Lorentz violation [1–3]. One approach to incorporate Lorentz and CPT violation into a Lorentz-symmetric Lagrangian by adding explicit breaking terms [4,5]. The resulting field theories should be regarded as effective theories arising from the more fundamental theory. Problems with microcausality are addressed in the underlying fundamental theory at the energy scales at which the effective theory breaks down [6]. The experimental implications of Lorentz and CPT violation parameterized in this manner have been explored extensively in recent years [7].

The first supersymmetric model with Lorentz and CPT violation involved extending [1] the Wess-Zumino model [8]. Two extensions were found, and these two models admit a superspace formulation [2].

II. THE WESS-ZUMINO LAGRANGIAN

A useful tool for developing supersymmetric field theories is to represent a supermultiplet of component fields as a superfield defined over a superspace of coordinates

$$z^M = (x^\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}) \tag{1}$$

The four anticommuting coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$ form two-component Weyl spinors. A superfield $\Phi(x, \theta, \bar{\theta})$ is then a function of the commuting spacetime coordinates x^μ and of four anticommuting coordinates θ^α and $\bar{\theta}_{\dot{\alpha}}$ which form two-component Weyl spinors. A chiral superfield is a function of $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$ and θ, i.e.

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \phi(y) + \sqrt{2}\theta\psi(y) + (\theta\theta)\mathcal{F}(y) , \\ &= \phi(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) + i\sqrt{2}\theta\sigma^\mu\bar{\theta}\theta\partial_\mu\psi(x) + (\theta\theta)\mathcal{F}(x) , \end{aligned} \tag{2}$$

where one can define the usual real components of the complex scalar components as

$$\phi = \tfrac{1}{\sqrt{2}}(A + iB), \quad \mathcal{F} = \tfrac{1}{\sqrt{2}}(F - iG). \tag{3}$$

The conjugate superfield is

$$\Phi^*(x, \theta, \bar{\theta}) = \phi^*(z) + \sqrt{2}\bar{\theta}\bar{\psi}(z) + (\bar{\theta}\bar{\theta})\mathcal{F}^*(z) \ , \tag{4}$$

where $z^\mu = y^{\mu*} = x^\mu - i\theta\sigma^\mu\bar{\theta}$. The Wess-Zumino Lagrangian can now be derived from the superspace integral [9]

$$\int d^4\theta \Phi^*\Phi + \int d^2\theta \left[\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c.\right] \ . \tag{5}$$

The superspace integral over $\int d^4\theta$ projects out the $(\theta\theta)(\bar{\theta}\bar{\theta})$ component of the $\Phi^*\Phi$ superfield while the $\int d^2\theta$ projects out the $\theta\theta$ component of the superpotential. The result

$$\begin{aligned}
\mathcal{L}_{WZ} = {} & \partial_\mu\phi^*\partial^\mu\phi + \frac{i}{2}[(\partial_\mu\psi)\sigma^\mu\bar{\psi} + (\partial_\mu\bar{\psi})\bar{\sigma}^\mu\psi] + \mathcal{F}^*\mathcal{F} \\
& + m\left[\phi\mathcal{F} + \phi^*\mathcal{F}^* - \tfrac{1}{2}\psi\psi - \tfrac{1}{2}\bar{\psi}\bar{\psi}\right] \\
& + g\left[\phi^2\mathcal{F} + \phi^{*2}\mathcal{F}^* - \phi(\psi\psi) - \phi^*(\bar{\psi}\bar{\psi})\right] \ ,
\end{aligned} \tag{6}$$

is a Lagrangian which transforms into itself plus a total derivative under a supersymmetric transformation.

One can formulate the Wess-Zumino model in terms of differential operators that act on the superfields. Define

$$X \equiv (\theta\sigma^\mu\bar{\theta})\partial_\mu \ , \tag{7}$$

so that

$$U_x \equiv e^{iX} = 1 + i(\theta\sigma^\mu\bar{\theta})\partial_\mu - \frac{1}{4}(\theta\theta)(\bar{\theta}\bar{\theta})\Box \ . \tag{8}$$

The expansion terminates because of the anticommuting nature of θ and $\bar{\theta}$. Since X is a derivative operator, the action of U_x on a superfield $\mathcal{S}$ is a coordinate shift in which the spacetime coordinate x^μ is shifted to y^μ,

$$U_x\mathcal{S}(x, \theta, \bar{\theta}) = \mathcal{S}(y, \theta, \bar{\theta}) \ . \tag{9}$$

The chiral superfield $\Phi(x, \theta, \bar{\theta})$ is a function of y^μ and θ only, so it must then be of the form $\Phi(x, \theta, \bar{\theta}) = U_x\Psi(x, \theta)$ for some function Ψ. The kinetic terms of the Wess-Zumino model can be expressed as

$$\int d^4\theta \left[U_x^*\Psi(x, \bar{\theta})^*\right]\left[U_x\Psi(x, \theta)\right] = \int d^4\theta \Phi^*(z, \bar{\theta})\Phi(y, \theta) \ . \tag{10}$$

III. LORENTZ VIOLATION

It has been shown on quite general grounds that CPT violation implies Lorentz violation [10,11], but it is possible to have Lorentz violation without CPT violation.. Of the two extensions to the Wess-Zumino model, the first does not contain CPT violation and it is

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easily understood as arising from the following substitution $\partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu$, where $k_{\mu\nu}$ is a real, symmetric, traceless, and dimensionless coefficient responsible for the Lorentz violation. so that

$$\mathcal{L}_{\text{Lorentz}} = \mathcal{L}_{WZ}(\partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu) . \tag{11}$$

The coefficient $k_{\mu\nu}$ transforms as a 2-tensor under observer Lorentz transformations but as a scalar under particle Lorentz transformations[1]. Adding Lorentz violation in this fashion can be immediately extended to encompass supersymmetric gauge theories as well since the superfield formulations of those theories will carry forward under the substitution $\partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu$.

The second extension of the Wess-Zumino model violates CPT in addition to containing Lorentz violation and is given by the Lagrangian [1]

$$\mathcal{L}_{\text{CPT}} = [(\partial_\mu - ik_\mu)\phi^*] [(\partial^\mu + ik^\mu)\phi]$$
$$+ \frac{i}{2}[((\partial_\mu + ik_\mu)\psi)\sigma^\mu\bar\psi + ((\partial_\mu - ik_\mu)\bar\psi)\bar\sigma^\mu\psi] + \mathcal{F}^*\mathcal{F} . \tag{12}$$

Here the Lorentz and CPT violation is controlled by k_μ, which is a real coefficient of mass dimension one which transforms as a four-vector under observer Lorentz transformations but remains unaffected by particle Lorentz transformations [4,5]. Unlike the coefficient $k_{\mu\nu}$, the quantity k_μ has an odd number of four-indices so it violates CPT. The Lagrangian for the model with the CPT-violating coefficient k_μ can be obtained from the kinetic part of the Wess-Zumino Lagrangian in Eqn. (6) with the appropriate substitutions $\partial_\mu \to \partial_\mu \pm ik_\mu$.

The two Lorentz-violating models can be described in the superspace formalism in a way that parallels that of the ordinary Wess-Zumino model. Define superfields

$$\Phi_y(x, \theta, \bar\theta) = \Phi(x, \theta, \bar\theta; \partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu)$$
$$= \phi(x_+) + \sqrt{2}\theta\psi(x_+) + (\theta\theta)F(x_+) , \tag{13}$$

and

$$\Phi_y^*(x, \theta, \bar\theta) = \Phi^*(x, \theta, \bar\theta; \partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu)$$
$$= \phi^*(x_-) + \sqrt{2}\bar\theta\bar\psi(x_-) + (\bar\theta\bar\theta)F^*(x_-) , \tag{14}$$

where

$$x_\pm^\mu = x^\mu \pm i\theta\sigma^\mu\bar\theta \pm ik^{\mu\nu}\theta\sigma_\nu\bar\theta ., \tag{15}$$

are shifted coordinates that take the place of y^μ and z^μ. Under a CPT-transformation the chiral superfield Φ_y and the antichiral superfield Φ_y^* transform into themselves just as the usual superfields Φ and Φ^* do. The Lagrangian in Eqn. (11) can be obtained by the same

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[1]The terms containing $k_{\mu\nu}$ give rise to Lorentz violation, the physics remains independent of the particular coordinate system that is used to describe it.

superspace integral in Eqn. (5) with the superfields Φ_y and Φ_y^* substituted in the place of Φ and Φ^* (see Eqn. (24) below).

The appropriate modified superfields for the model in Eqn. (12),

$$\Phi_k(x, \theta, \bar{\theta}) = \Phi(x, \theta, \bar{\theta}; \partial_\mu \to \partial_\mu + ik_\mu) \ , \tag{16}$$

and

$$\Phi_k^*(x, \theta, \bar{\theta}) = \Phi^*(x, \theta, \bar{\theta}; \partial_\mu \to \partial_\mu - ik_\mu) \ , \tag{17}$$

cannot be obtained from a coordinate shift, but can be obtained from a more general superspace transformation.

IV. SUPERSPACE TRANSFORMATIONS

The Lorentz-violating extensions of the Wess-Zumino model can be understood as transformations on the superfields. Define

$$Y \equiv k_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu \ , \tag{18}$$
$$K \equiv k_\mu(\theta\sigma^\mu\bar{\theta}) \ , \tag{19}$$

so that

$$U_y \equiv e^{iY} = 1 + ik_{\mu\nu}(\theta\sigma^\mu\bar{\theta})\partial^\nu - \frac{1}{4}k_{\mu\nu}k^{\mu\rho}(\theta\theta)(\bar{\theta}\bar{\theta})\partial^\nu\partial_\rho \ , \tag{20}$$

$$T_k \equiv e^{-K} = 1 - k_\mu(\theta\sigma^\mu\bar{\theta}) + \frac{k^2}{4}(\theta\theta)(\bar{\theta}\bar{\theta}) \ . \tag{21}$$

Since Y, like X, is a derivative operator, the action of U_y on a superfield $\mathcal{S}$ is a coordinate shift. The appearance of terms of order $\mathcal{O}(k^2)$ in both cases is easily understood in terms of these operators. Furthermore we have $U_y^* = U_y^{-1}$ while $T_k^* = T_k$ and not its inverse.

The supersymmetric models with Lorentz-violating terms can be expressed in terms of new superfields,

$$\Phi_y(x, \theta, \bar{\theta}) = U_y U_x \Psi(x, \theta) \ , \tag{22}$$
$$\Phi_y^*(x, \theta, \bar{\theta}) = U_y^{-1} U_x^{-1} \Psi^*(x, \bar{\theta}) \ . \tag{23}$$

Applying U_y to the chiral and antichiral superfields merely effects the substitution $\partial_\mu \to \partial_\mu + k_{\mu\nu}\partial^\nu$. Since U_y involves a derivative operator just as U_x, the derivation of the chiral superfield Φ_y is a function of the variables x_+^μ and θ analogous to how, in the usual case, Φ is a function of the variables y^μ and θ. The Lagrangian is given by

$$\int d^4\theta \Phi_y^* \Phi_y + \int d^2\theta \left[\frac{1}{2}m\Phi_y^2 + \frac{1}{3}g\Phi_y^3 + h.c.\right]$$
$$= \int d^4\theta \left[U_y^*\Phi^*\right][U_y\Phi] + \int d^2\theta \left[\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + h.c.\right] \ . \tag{24}$$

For the CPT-violating model the superfields have the form

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$$\Phi_k(x, \theta, \bar{\theta}) = T_k U_x \Psi(x, \theta) \,, \tag{25}$$

$$\Phi_k^*(x, \theta, \bar{\theta}) = T_k U_x^{-1} \Psi^*(x, \bar{\theta}) \,. \tag{26}$$

It is helpful to note that the transformation U_x acts on Ψ and its inverse U_x^{-1} acts on Ψ^*, while the same transformation T_k acts on both Ψ and Ψ^* (since $T_k^* = T_k$). A consequence of this fact is that the supersymmetry transformation will act differently on the components of the chiral superfield and its conjugate. Specifically the chiral superfield Φ_k is the same as Φ with the substitution $\partial_\mu \to \partial_\mu + ik_\mu$ whereas the antichiral superfield Φ_k^* is the same as Φ^* with the substitution $\partial_\mu \to \partial_\mu - ik_\mu$, as in Eqns. (16) and (17).

The CPT-violating model in Eqn. (12) can then be represented in the following way as a superspace integral:

$$\int d^4\theta \Phi_k^* \Phi_k = \int d^4\theta \Phi^* e^{-2K} \Phi \tag{27}$$

Unlike the CPT-conserving model, the $(\theta\theta)(\bar{\theta}\bar{\theta})$ component of $\Phi^*\Phi$ no longer transforms into a total derivative. A specific combination of components of $\Phi^*\Phi$ does transform into a total derivative, and this combination is in fact the $(\theta\theta)(\bar{\theta}\bar{\theta})$ component of $\Phi_k^*\Phi_k$.

V. CONCLUSIONS

The Wess-Zumino model can be described in terms of superspace transformations and the projections of components of functions of the superfield. It was shown that the two Lorentz-violating extensions of the Wess-Zumino model can be understood in terms of analogous transformations on modified superfields and projections arising from superspace integrals.

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